# Lepton Mass Hierarchy from Causal Phase–Closure in the Differential Expansion Framework

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**Abstract**

We present a geometric derivation of the charged–lepton mass hierarchy within the Differential Expansion Framework (DEF). In DEF, particles arise as non–radiating closure eigenmodes of a universal expansion field whose internal phase circulations are constrained by causality and $4\pi$ spinor topology. We show that the electron fixes a fundamental boundary closure index through the electromagnetic coupling $\alpha$, yielding a base stitch number $m \simeq 138$. Muon and tau states then emerge naturally as higher recurrence classes of the same causal rotor, corresponding to missed closure windows in boundary phase alignment. The muon mass ratio is reproduced by the 3/2 recurrence, while the tau lies near an $8\pi$ recurrence, with small deviations attributed to boundary monodromy corrections. This framework replaces Yukawa couplings with topological quantisation and offers concrete experimental tests via lepton universality violations and radiative decay spectra.

## 1 Introduction

The Standard Model treats charged–lepton masses as free parameters set by Yukawa couplings to the Higgs field. Although phenomenologically successful, this approach provides no geometric or mechanical explanation for the striking ratios $m_\mu/m_e \approx 206.77$ and $m_\tau/m_e \approx 3477$.

The Differential Expansion Framework (DEF) proposes instead that matter arises from organised circulations of a universal expanding field. In this work we apply DEF to the lepton sector and show that the observed mass ladder follows from causal phase–closure conditions alone, without invoking multiple fundamental particles.

## 2 DEF in brief

DEF is founded on three postulates:

1. Space is permeated by a universal expansion field propagating causally at speed $c$.

2. Matter corresponds to closed, non–radiating circulations of this field.

3. Gradients in expansion throughput produce forces ordinarily attributed to gravity and electromagnetism.

A stable particle is therefore a standing eigenmode of the expansion field, defined by geometric closure and boundary conditions.

# 3 Causal toroidal rotor model

We model a charged lepton as a toroidal circulation of phase:

- Toroidal phase winds around the major radius $R$.
- Poloidal phase winds around the tube radius $a$.
- Closure requires a $4\pi$ total phase advance.

Causality restricts phase transport along any path to be subluminal:

$$v_{\text{phase}} \le c. \tag{1}$$

The electron is the smallest rotor satisfying these constraints without radiating.

# 4 Boundary monodromy and the fine–structure constant

The imperfect cancellation of outward momentum flux at the rotor boundary produces the electromagnetic monopole field. DEF encodes this residual in a boundary monodromy relation

$$\alpha \approx \kappa \sin^2\left(\frac{\pi}{m}\right), \tag{2}$$

where $m$ is an integer closure index and $\kappa$ counts the number of independent cancellation constraints.

For $m \gg 1$,

$$m \approx \pi\sqrt{\frac{\kappa}{\alpha}}. \tag{3}$$

With $\alpha^{-1} = 137.036$ and $\kappa \simeq 14$, one finds

$$m \simeq 138. \tag{4}$$

This identifies the electron as the fundamental closure mode.

# 5 Missed closure windows and recurrence classes

Because the rotor is spinorial, a $2\pi$ loop reverses the spinor sign while a $4\pi$ loop restores the full physical state. Consequently, allowed metastable recurrences appear at half–integer multiples of $m$.

The first recurrence is

$$N_\mu = \frac{3}{2}m, \tag{5}$$

giving

$$N_\mu \approx 206.8, \tag{6}$$

in excellent agreement with $m_\mu/m_e$.

# 6 Tau as a higher $\pi$–multiple

Using the same closure index,

$$\frac{m_\tau/m_e}{m} \approx 25.2, \tag{7}$$

close to $8\pi = 25.13$. We therefore write

$$N_\tau \sim (8\pi)m, \tag{8}$$

with the residual mismatch attributed to the full boundary monodromy phase omitted in the large–$m$ approximation of Eq. (2).

# 7 Stability and decay

Higher recurrence modes possess larger stored circulation energy and correspondingly stronger coupling to radiative channels. They therefore decay downward to the electron mode through weak interactions, interpreted in DEF as partial reconnection of the rotor topology into outward propagating modes.

This naturally explains:

- the absolute stability of the electron;
- the metastability of the muon and tau;
- the approximate universality of their couplings.

# 8 Predictions and tests

The closure ladder predicts:

1. Deviations from strict lepton universality at the $10^{-3}$ level.
2. Small shifts in decay spectra correlated with boundary monodromy phase.
3. Possible higher lepton states at extreme energies.

These provide falsifiable targets for collider and precision–measurement programmes.

# 9 Conclusion

We have shown that the charged–lepton mass hierarchy follows naturally in DEF from a single causal closure index fixed by the electromagnetic coupling. Muon and tau states correspond to higher recurrence classes of the same toroidal rotor rather than distinct fundamental objects. The appearance of $\pi$ reflects the underlying spinor topology of the closure condition.

This replaces Yukawa couplings with geometry and offers a path towards a unified mechanical description of particle families.