Derivation of Hawking Radiation in the Differential Expansion Framework

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Abstract

The Differential Expansion Framework (DEF) models gravity as a gradient in an expansion field $E = c + e$, where e is a local deviation and $\sigma = \frac{4\pi G}{c^2}$. We derive Hawking radiation for a DEF black hole, showing it matches the general relativity (GR) result for large black holes while offering insights via the non-singular Sikora limit ($E = 0$ at $r_S = \frac{GM}{c^2}$). The derivation uses the Unruh analogy and quantum field theory, highlighting DEF's energy conservation and potential modifications for micro-black holes.

1 Introduction

In general relativity (GR), Hawking radiation arises from quantum field effects near a black hole horizon, characterized by a temperature $T_H = \frac{\hbar c^3}{8\pi GMk_B}$. The Differential Expansion Framework (DEF) describes gravity via an expansion field $E = c + e$, with a non-singular core at the Sikora radius $r_S = \frac{GM}{c^2}$. We derive the Hawking temperature in DEF, emphasizing its consistency with GR and unique implications.

2 DEF Black Hole Metric

The DEF Schwarzschild metric (for $r > r_S$) is:

$$ds^2 = -\left(1 - \frac{\sigma M}{2\pi r}\right)c^2dt^2 + \left(1 - \frac{\sigma M}{2\pi r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $\sigma = \frac{4\pi G}{c^2} \approx 9.33 \times 10^{-27}$ m kg^{-1}, and the event horizon is at $r_s = \frac{\sigma M}{2\pi} = \frac{2GM}{c^2}$. For $r \leq r_S$, $e = -c$, implying $g_{00} = +1$, forming a non-singular core.

3 Surface Gravity and Unruh Analogy

Near the horizon ($r \approx r_s$), the metric resembles a Rindler spacetime. The surface gravity κ is:

$$\kappa = \frac{c^4}{4GM} = \frac{2\pi c^2}{\sigma M}. \tag{2}$$

The Unruh temperature for an accelerated observer is:

$$T = \frac{\hbar\kappa}{2\pi k_B} = \frac{\hbar c^4}{8\pi GMk_B} = \frac{\hbar c}{2\sigma Mk_B}. \tag{3}$$

Redshifting to infinity yields the Hawking temperature $T_H = T$.

4 Quantum Field Theory Derivation

Consider a massless scalar field ϕ satisfying $\Box_g\phi = 0$. In null coordinates ($u = ct - r^*$, $v = ct + r^*$), modes mix near the horizon via Bogoliubov transformations, producing a thermal spectrum:

$$\langle N_\omega \rangle = \frac{1}{e^{\hbar\omega/(k_B T_H)} - 1}, \tag{4}$$

with $T_H = \frac{\hbar c^4}{8\pi GMk_B}$.

5 DEF Interpretation

In DEF, radiation converts suppressed expansion energy (E_{field}) into particles ($E_{\text{radiation}}$), per $E_{\text{total}} = E_{\text{field}} + E_{\text{matter}} + E_{\text{radiation}}$. For large black holes ($M \gg m_P$), the Sikora core ($r_S \ll r_s$) has negligible effect. For micro-black holes, the core may halt evaporation at $M \sim m_P$.

6 Conclusion

DEF predicts Hawking radiation identical to GR for large black holes, with $T_H = \frac{\hbar c^3}{8\pi GMk_B}$. The Sikora limit suggests modified evaporation endpoints, preserving information in non-singular cores.