Derivation of the Schwarzschild Metric in the Differential Expansion Framework

1 Introduction

The Differential Expansion Framework (DEF) models gravity as a gradient in an expansion field $E = c + e$, where e is the local deviation (m s^{-1}) and $\sigma = \frac{4\pi G}{c^2}$. We derive the Schwarzschild metric, aligning with general relativity in the strong-field regime.

2 Weak-Field Limit

In the weak field:

$$\Phi = ce, \quad \nabla^2 e = \sigma c \rho, \quad \sigma = \frac{4\pi G}{c^2}$$

For a point mass M:

$$e(r) = -\frac{\sigma c M}{4\pi r} = -\frac{GM}{cr}, \quad \Phi = -\frac{GM}{r}$$

$$\mathbf{a} = -c\nabla e = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

This matches Newtonian gravity.

3 Metric Ansatz

Assume a static, spherically symmetric metric:

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Given $\Phi = ce$, and in the weak field $g_{00} \approx -\left(1 + \frac{2\Phi}{c^2}\right)$:

$$\frac{\Phi}{c^2} = \frac{e}{c}, \quad A(r) = 1 + \frac{2e}{c} = 1 - \frac{2GM}{c^2 r}$$

Assume:

$$B(r) = \left(1 - \frac{2GM}{c^2 r}\right)^{-1}$$

4 Schwarzschild Radius

$$r_s = \frac{\sigma M}{2\pi} = \frac{\left(\frac{4\pi G}{c^2}\right)M}{2\pi} = \frac{2GM}{c^2}$$

$$1 - \frac{2GM}{c^2 r} = 1 - \frac{\sigma M}{2\pi r}, \quad e = -\frac{\sigma c M}{4\pi r}$$

$$1 + \frac{2e}{c} = 1 - \frac{\sigma M}{2\pi r}$$

5 Final Metric

$$\boxed{ds^2 = -\left(1 - \frac{\sigma M}{2\pi r}\right) c^2 dt^2 + \left(1 - \frac{\sigma M}{2\pi r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)}$$

Or, using e:

$$\boxed{ds^2 = -\left(1 + \frac{2e}{c}\right) c^2 dt^2 + \left(1 + \frac{2e}{c}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)}$$

6 Conclusion

The DEF metric matches the Schwarzschild metric, reproducing Newtonian gravity in the weak field and GR's strong-field predictions, with $\sigma = (9.331988218 \pm 0.0002096873) \times 10^{-27}$ m kg^{-1}.